Rudi Mathematici

## $x^{4}-8200 x^{3}+25213040 x^{2}-34452464000 x+17652769695744=0$



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"Good morning, and welcome to The Wonders of Physics."


## January



## USAMO 1997 - Problem 1

Let $p_{1}, p_{2}, p_{3}, \ldots$ be the prime numbers listed in increasing order, and let $X_{0}$ be a real number between 0 and 1. For positive integer $k$, define:

$$
x_{k}= \begin{cases}0 & \text { if } x_{k-1}=0 \\ \left\{\frac{p_{k}}{x_{k-1}}\right\} & \text { if } x_{k-1} \neq 0\end{cases}
$$

where $\{x\}$ denotes the fractional part of $X$. Find, with proof, all $X_{0}$ satisfying $0<x_{0}<1$ for which the sequence $X_{0}, X_{1}, X_{2}, \ldots$ eventually becomes 0

Why Astronomy is better than Sex:
If you get tired, wait ten minutes and try it again.

## Statisticians

What do you get when you cross a statistician with a chiropractor?

You get an adjusted $\mathbf{R}$ squared from
a BACKward regression problem.
"I know not what I appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell, whilst the great ocean of truth lay all undiscovered before me"

Isaac NEWTON
"The proof of the Hilbert Basis Theorem is not mathematics; it is theology."

Camille JORDAN
"It's very good jam," said the Queen.
"Well, I don't want any to-day, at any rate."
"You couldn't have it if you did want it," the Queen said. "The rule is jam tomorrow and jam yesterday but never jam to-day."
"It must come sometimes to "jam to-day," Alice objected.
"No it can't," said the Queen. "It's jam every other day; to-day isn't any other day, you know."
"I don't understand you," said Alice. "It's dreadfully confusing.'

Charles DOGSON
"Mathematics is a game played according to certain simple rules with meaningless marks on paper."

David HILBERT
"A mathematician's reputation rests on the number of bad proofs he has given"

Abram BESICOVITCH

## February



## USAMO 1997 - Problem 2

Let $A B C$ be a triangle, and draw isosceles triangles $B C D, C A E, A B F$ externally to $A B C$, with $B C, C A, A B$ as their respective bases. Prove that the lines through
$A, B, C$ perpendicular to the lines $\overline{E F}$,
$\overline{F D}, \overline{D E}$, respectively, are concurrent.
Why Astronomy is better than Sex:
Nobody cares if you are ugly.

## Statisticians

A statistician is someone who is skilled at drawing a precise line from an unwarranted assumption to a foregone conclusion.
"Common sense is not really so common."
Antoine ARNAUD
"Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not. "Immortality" may-be a silly word, but probably a mathematician "has the best chance of whatever it may mean."

Godfried HARDY
"it would be better for the true physics if there were no mathematicians on earth"

Daniel BERNOULLI
"Epur si muove"
Galileo GALILEI
...an incorrect theory, even if it cannot be inhibited by any contradiction that would refute it, is none the less incorrect, just as a criminal policy is none the less criminal even if it cannot be inhibited by any court that would curb it.

Luitzen BROUWER
"Euler calculated without effort, just as men breathe, as eagles sustain themselves in the air"

Dominique ARAGO

## March



USAMO 1997 - Problem 3
Prove that for any integer $n$, there exists an unique polynomial $Q$ with coefficients in $\{1,2,3, \ldots, 9\}$ such that $Q(-2)=Q(-5)=n$.
Why Astronomy is better than Sex:
Forty years from now, you can still participate regularly.
Why pi is inferior to e
You can't confuse e with a food product.
Why e is inferior to pi
e is less challenging to spell than pi.
"And what are these fluxions? The velocities of evanescent increments? They are neither finite quantities, nor quantities infinitely small, nor yet nothing. May we not call them ghosts of departed quantities?"

George BERKELEY
"Common sense is nothing more than a deposit of prejudices laid down in the mind before you reach eighteen."

Albert EINSTEIN
*We [he and Halmos] share a philosophy about linear algebra; we think basis-free, we write basis-free, but when the chips are down we close the office door and compute with matrices like fury."

Irving KAPLANSKY
The mathematician's best work is art, a high perfect art, as daring as the most secret dreams of imagination, clear and limpid. Mathematical genius and artistic genius touch one another.

Gosta MITTAG-LEFFLER
"A Mathematician is a machine for turning coffee into theorems. "

Paul ERDOS
The profound study of nature is the most fertile source of mathematical discoveries.

Jean Baptiste FOURIER
"What we know is not much. What we do not know is immense."

Pierre Simon de LAPLACE

## April



## USAMO 1997 - Problem 4

To clip a convex $n$-agon means to choose a pair of consecutive sides $A B, B C$ and to replace them by the three segments $A M$, $M N$ and $N C$, where $M$ is the midpoint of $A B$ and $N$ is the midpoint of $B C$. In other words, one cuts off the triangle $M B N$ to obtain a convex $(n+1)$-agon. A regular hexagon $P_{6}$ of area 1 is clipped to obtain an heptagon $P_{7}$. Then $P_{7}$ is clipped (in one of the seven possible ways) to obtain an octagon $P_{8}$, and so on. Prove that no matter how the clippings are done, the area of $P_{n}$ is greater than $1 / 3$, for all $n \geq 6$
Why Astronomy is better than Sex:
Doesn't matter if kids hear you moaning, oohing and aahing.

## Why pi is inferior to e

You don't need to know Greek to be able to use e.
Why e is inferior to pi
$\mathrm{e} \sim=2.718281828459045$, which can be easily memorized to its billionth place, whereas pi needs "skills" to be memorized.
"Knowing what is big and what is small is more important than being able to solve partial differential equations"

Stanislaw Marcin ULAM
"You treat world history as a mathematician does mathematics, in which nothing but laws and formulae exist, no reality, no good and evil, no time, no yesterday, no tomorrow, nothing but an eternal shallow, mathematical present."

Otto Ludwig HESSE
"An important scientific innovation rarely makes its way by gradually winning over and converting its opponents: it rarely happens that Saul becomes Paul. What does happen is that its opponents gradually die out, and that the growing generation is familiarised with the ideas from the beginning"

## Max Karl Ernst Ludwig PLANCK

"Everyone knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions."

Felix KLEIN
"The fact that the author thinks slowly is not serious, but the fact that he publishes faster than he thinks is inexcusable."

Wolfgang PAULI

## May



USAMO 1997 - Problem 5
Prove that, for all positive real numbers $a$, $b, c$ :

$$
\begin{aligned}
& \left(a^{3}+b^{3}+a b c\right)^{-1}+ \\
& \left(b^{3}+c^{3}+a b c\right)^{-1}+ \\
& \left(c^{3}+a^{3}+a b c\right)^{-1} \leq(a b c)^{-1}
\end{aligned}
$$

Why Astronomy is better than Sex:
You can do it all night.
Why pi is inferior to e
e stands for Euler's Number, pi doesn't stand for squat.
Why e is inferior to pi
The character for e is so cheap that it can be found on a keyboard. But PI is special (it's under "special symbols"-in word processor programs.)
"Nature is not embarrassed by difficulties of analysis."

Augustin Jean FRESNEL
"Now one may ask, "What is mathematics doing in a physics lecture? "We have several possible excuses: first, of course, mathematics is an important tool, but that would only excuse us for giving the formula in two minutes. On the other hand, in theoretical physics we discover that all our laws can be written in mathematical form; and that this has a certain simplicity and beauty about it. But the real reason is that the subject is enjoyable, and although we humans cut nature up in different ways, and we have different courses in different departments, such compartmentalization is really artificial, and we should take our intellectual pleasures where we find them.

Richard Phillips FEYNMAN
"To isolate mathematics from the practical demands of the sciences is to invite the sterility of a cow shut away from the bulls."

Pafnuti Lvovi CHEBYSHEV
"Mathematics is very much like poetry. What makes a great poem is tat there is a great amount of thought expressed in very few words. in this sense, formulas like $\mathrm{e}^{\pi_{i}}+1=0$ are poems.

Lipa BERS

## June



## USAMO 1997 - Problem 6

Suppose the sequence of non-negative integers $a_{1}, a_{2}, \ldots, a_{1997}$ satisfies

$$
a_{i}+a_{j} \leq a_{i+j} \leq a_{i}+a_{j}+1
$$

for all $i, j \geq 1$ with $i+j \leq 1997$. Show that there exists a real number $x$ such that $a_{n}=\lfloor n x\rfloor$ for all $1 \leq n \leq 1997$.

## Why Astronomy is better than Sex:

Less guilt the next morning.
Why pi is inferior to e
$e$ is easier to spell than pi.
Why e is inferior to pi
To read pi, you don't have to know that Euler's name is really pronounced Oiler.
"It can be of no practical use to know that $\pi$ is irrational, but if we can know, it surely would be intolerable not to know".

Edward Charles TICHMARSH
The mathematical education of the young physicist [Albert Einstein] was not very solid, which I am in a good position to evaluate since he obtained it from me in Zurich some time ago.

Hermann MINKOWSKY
"What I give form to in daylight is only one per cent of what I have seen in darkness"

Maurits Cornelius ESCHER
Life is good for only two things, discovering mathematics and teaching mathematics.

Simeon Denis POISSON
"The more I see of men, the better I like my dog"

Blaise PASCAL
"Science is a differential equation. Religion is a boundary condition"

Alan Mathison TURING
"In my opinion, a mathematician, in so far as he is a mathematician, need not preoccupy himself with philosophy .- an opinion, moreover, which has been expressed by many philosophers."

Henri LEBESGUE

| 27 | 1 | T | (1643) Gottfried Wilhelm von LEIBNIZ (1788) Jean Victor PONCELET |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 2 | W | (1820) William John Racquorn RANKINE (1852) William BURNSIDE |  |
|  | 3 | T | (1807) Ernest Jean Philippe Fauque de JONQUIERE (1897) Jesse DOUGLAS |  |
|  | 4 | F | (1906) Daniel Edwin RUTHERFORD <br> (1917) Michail Samuilovich LIVSIC |  |
|  | 5 | S | (1936) James MIRRLEES |  |
|  | 6 | S | (1849) Alfred Bray KEMPE |  |
| 28 | 7891011111 | M | (1816) Johann Rudolf WOLF <br> (1906) William FELLER <br> (1922) Vladimir Aleksandrovich MARCHENKO |  |
|  |  | T | (1760) Christian KRAMP |  |
|  |  | W | (1845) George Howard DARWIN - | t |
|  |  | T | (1862) Roger COTES <br> (1868) Oliver Dimon KELLOGG |  |
|  |  | F | (1857) Sir Joseph LARMOR <br> (1890) Giacomo ALBANESE |  |
|  |  | S | (1875) Ernest Sigismund FISCHER <br> (1895) Richard BUCKMINSTER FULLER |  |
|  |  | S | (1527) John DEE <br> (1741) Karl Friedrich HINDENBURG |  |
| 29 | 14 M | M | (1671) Jacques S'ALLONVILLE (1793) George GREEN |  |
|  | 15 | T | (1865) Wilhelm WIRTINGER (1906) Adolph Andrej Pavlovich YUSHKEVICH |  |
|  | 16 | W | (1678) Jakob HERMANN <br> (1903) Irmgard FLUGGE-LOTZ |  |
|  | 17 | T | (1831) Victor Mayer Amedee' MANNHEIM <br> (1837) Wilhelm LEXIS |  |
|  | 18 | F | (1013) Hermann von REICHENAU <br> (1635) Robert HOOKE <br> (1853) Hendrich Antoon LORENTZ |  |
|  | 19 | S | (1768) Francois Joseph SERVOIS |  |
|  | 20 | S | (1876) Otto BLUMENTHAL <br> (1947) Gerd BINNIG |  |
| 30 | 21 | M | (1620) Jean PICARD <br> (1848) Emil WEYR <br> (1849) Robert Simpson WOODWARD |  |
|  | 22 |  | (1784) Friedrich Wilhelm BESSEL |  |
|  | 23 | W | (1775) Etienne Louis MALUS | $\square$ |
|  | 24 |  | (1851) Friedrich Herman SCHOTTKY <br> (1871) Paul EPSTEIN <br> (1923) Christine Mary HAMILL |  |
|  | 25 | F | (1808) Johanh Benedict LISTING |  |
|  | 26 |  | (1903) Kurt MAHLER |  |
|  |  |  | (1667) Johann BERNOULLI <br> (1801) George Biddel AIRY <br> (1848) Lorand Baron von EOTVOS <br> (1871) Ernst Friedrich Ferdinand ZERMELO |  |
| 31 | 28 | M | (1954) Gerd FALTINGS |  |
|  |  |  | (1898) Isidor Isaac RABI |  |
|  | 30 | W | (1889) Vladimir Kosma ZWORKYN |  |
|  |  | T | (1704) Gabriel CRAMER (1712) Johann Samuel KOENIG |  |

## USAMO 1998 - Problem 1

Suppose that the set $\{1,2, \ldots, 1998\}$ has been partitioned into disjoint pairs $\left\{a_{i}, b_{i}\right\}$ $(1 \leq i \leq 999)$ so that for all $i,\left|a_{i}-b_{i}\right|$ equals 1 or 6 . Prove that the sum
$\left|a_{1}-b_{1}\right|+\left|a_{2}-b_{2}\right|+\ldots+\mid a_{999}-b_{999}$ ends in the digit 9.

## Why Astronomy is better than Sex:

You can experience multiple objects in a single session

## Why pi is inferior to e

pi $\sim=3.14$ while $\mathrm{e} \sim=2.718281828459045$
Why e is inferior to pi
Pi is much shorter and easier to say than "Euler's Number"
"When working on a problem, I never think about beauty; I think only of how to solve the problem. But when I have finished, if the solution is not beautiful, I know that it is wrong."

Richard Buckminster FULLER
" There is (gentle reader) nothing (the works of God only set apart) which so much beautifies and adorns the soul and mind of man as does knowledge of the good arts and sciences. . Many ... arts there are which beautify the mind of man; but of all none do more garnish and beautify it than those arts which are called mathematical, unto the knowledge of which no man can attain, without perfect knowledge and instruction of the principles, grounds, and Elements of Geometry."

John DEE
"CEIIOSSOTTUU
Anagram to establish priority in the discovery of elasticity: "Ut tensio, sic uis"

Robert HOOKE
"[The infinitesimals] neither have nor can
have theory; in practise it is a dangerous instrument in the hands of beginners ... anticipating, for my part, the judgement of posterity, I would predict that this method will be accused one day, and rightly, of having retarded the progress of the mathematical sciences. "

Francois Joseph SERVOIS
"A quantity which is increased or decreased by an infinitely small quantity is neither increased nor decreased."

Johann BERNOULLI

## August



## USAMO 1998 - Problem 2

Let $C_{1}$ and $C_{2}$ be concentric circles, with $C_{2}$ in the interior of $C_{1}$. From a point $A$ of $C_{1}$ one draws the tangent $A B$ to $C_{2}$ ( $B \in C_{2}$ ). Let $C$ be the second point of intersection of $A B$ and $C_{1}$, and let $D$ the midpoint of $A B$. A line passing through $A$ intersects $C_{2}$ at $E$ and $F$ in such a way that the perpendicular bisectors of $D E$ and $C F$ intersect at a point $M$ on $A B$. Find, with proof, the ratio $A M / M C$.

## Why Astronomy is better than Sex:

Person you're with doesn't fantasize you're someone else.

## Why pi is inferior to e

The character for e can be found on a keyboard, but pi sure can't.
Why e is inferior to pi
e is named after a person, but pi stands for itself.
"The whole form of mathematical thinking was created by Euler. It is only with the greatest of difficulty that one is able to follow the writings of any author preceding Euler, because it was not yet known how to let the formulas speak for themselves. This art Euler was the first to teach."

Ferdinand RUDIO
"There are surely worse things than being wrong, and being dull and pedantic are surely among them.'
"This result is too beautiful to be false; it is more important to have beauty in one's equations than to have them fit experiment. "

Paul Adrien Maurice DIRAC
"And perhaps, posterity will thank me for having shown it that the ancients did not know everything."

Pierre de FERMAT
"Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos ejusdem nominis fas est dividere: cujus rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet"

Pierre de FERMAT
"Newton is, of course, the greatest of all Cambridge professors; he also happens to be the greatest disaster that ever befell not merely Cambridge mathematics in particular, but British mathematical science as a whole"

Leonard ROTH

## September



## USAMO 1998, Problem 3

Let $a_{0}, a_{1}, \ldots, a_{n}$ be numbers from the interval $(0, \pi / 2)$ such that

$$
\sum_{i=0}^{n} \tan \left(a_{i}-\frac{\pi}{4}\right) \geq n-1
$$

Prove that

$$
\prod_{i=0}^{n} \tan \left(a_{0}-\frac{\pi}{4}\right) \geq n^{n+1}
$$

Why Astronomy is better than Sex:
There is less shame when purchasing the equipment
Why pi is inferior to e
Everybody fights for their piece of the pie.
Why e is inferior to pi
People mistakenly confuse Euler's Number (e) with Euler's Constant (gamma). There is no confusion with the one and only PI
"The importance of the "New Mathematics" lies mainly in the fact that it has taught us the difference between the disc and the circle."

René THOM
"If it's just turning the crank it's algebra, but if it's got an idea in it, it's topology."

Solomon LEFSCHETZ
"This branch of mathematics [Probability] is the only one, I believe, in which good writers frequently get results which are entirely erroneous."

Charles Sanders PEIRCE
"We may as well cut out the group theory. That is a subject that will never be of any use in physics."
sir James Hopwood JEANS
"If error is corrected whenever it is recognised, the path of error is the path of truth."

Hans REICHENBACH
It is a matter for considerable regret that Fermat did not leave us with the proofs of the theorems he discovered. In truth, Messrs Euler and Lagrange have proved most of these theorems, and have even substituted extensive theories for the isolated propositions of Fermat. But there are several proofs which have resisted their efforts.

Adrien-Marie LEGENDRE
[Upon proving that the best betting strategy for "Gambler's Ruin" was to bet all on the first trial.]
"It is true that a man who does this is a fool. I have only proved that a man who does anything else is an even bigger fool."

Julian Lowell COOLIDGE

## October



## USAMO 1998, Problem 4

A computer screen shows a $98 \times 98$ chessboard, coloured in the usual way. One can select with a mouse any rectangle with sides on the lines of the chessboard and click the mouse button: as a result, the colours in the selected rectangle switch (black becomes white, white becomes black). Find, with proof, the minimum number of mouse clicks needed to make the chessboard all one colour.
Why Astronomy is better than Sex:
The telescope isn't going to make you pay child support for the next eighteen years.

## Why pi is inferior to e

$\ln \left(\mathrm{pi}^{\wedge} \wedge 1\right)$ is a really nasty number, but $\ln \left(e^{\wedge} 1\right)=1$

## Why e is inferior to pi

e you understand what it is even though you start learning it late when you're in precalculus. But pi, even after five or six years it's still hard to know what it really is.

An expert is a man who has made all the mistakes which can be made in a very narrow field"

Niels BOHR
" $2^{30}\left(2^{31}-1\right)$ is the greatest perfect number that will ever be discovered, for, as they are merely curious without being useful, it is not likely that any person will attempt to find a number beyond it"

Peter BARLOW
I recognize the lion by his paw.
[After reading an anonymous solution to a problem that he realized was Newton's solution.]

Jacob (II) BERNOULLI
"The Council of the Royal Society is a collection of men who elect each other to office and then dine together at the expense of this society to praise each other over wine and give each other medals."

## Charles BABBAGE

"Unfortunately what is little recognized is that the most worthwhile scientific books are those in which the author clearly indicates what he does not know; for an author most hurts his readers by concealing difficulties."

Evariste GALOIS
"It is true that a mathematician who is not also something of a poet will never be a perfect mathematician."

Karl Theodor Wilhelm WEIERSTRASS

## November



## USAMO 1998, Problem 5

Prove that for every $n \geq 2$, there is a set $S$ of $n$ integers such that $(a-b)^{2}$ divides $a b$ for every distinct $a, b \in S$.
Why Astronomy is better than Sex:
Guaranteed to get at least a little something in view.
Why pi is inferior to e
e is used in calculus while pi is used in baby geometry:-
Whye is inferior to pi
e has an easy limit definition and infinite series. The limit definition of pi and the infinite series are much harder.
"Of the many forms of false culture, a premature converse with abstractions is perhaps the most likely to prove fatal to the growth of a masculine vigour of intellect."

George BOOLE
"A scientist can hardly meet with anything more undesirable than to have the foundations give way just as the work is finished. I was put in this position by a letter from Mr. Bertrand Russell when the work was nearly through the press."

Fredrich Ludwig Gottlob FREGE
The history of astronomy is a history of receding horizons,

Edwin HUBBLE
"Logic is the hygiene the mathematician practices to keep his ideas healthy and strong."

Hermann Klaus Hugo WEYL
"The British Mathematical Colloquium consists of three days of mathematics with no dogs and no wives

John Henry Constantine WHITEHEAD
Algebra is generous: she often gives more than is asked for.

Jean D'ALEMBERT
"The modern physicist is a quantum theorist on Monday, Wednesday, and Friday and a student of gravitational relativity theory on Tuesday, Thursday, and Saturday. On Sunday he is neither, but is praying to his God that someone, preferably himself, will find the reconciliation between the two views."

Benoit MANDELBROT

## December



## USAMO 1998, Problem 6

Let $n \geq 5$ be an integer. Find the largest integer $k$ (as a function of $n$ ) such that there exists a convex $n$ - agon $A_{1} A_{2} \ldots A_{n}$ for which exactly $k$ of the quadrilaterals $A_{i} A_{i+1} A_{i+2} A_{i+3}$ have an inscribed circle. $\left(\right.$ Here $A_{n+j}=A_{j}$.)
Why Astronomy is better than Sex:
You don't have to compliment the person that gave you a view.
Why pi is inferior to e
'e' is the most commonly picked vowel in Wheel of Fortune.
Why e is inferior to pi
PI is the bigger piece of pie
"Die ganze Zahl schuf der liebe Gott, alles Übrige ist Menschenwerk."
= Leopold KRONECKER
"The shortest path between two truths in the real domain passes through the complex domain."

Jaques Salomon HADAMARD
"Now it is quite clear to me that there are no solid spheres in the heavens, and those that have been devised by authors to save the appearances, exist only in their imagination, for the purpose of permitting the mind to conceive the motion which the heavenly bodies trace in their courses."

Tycho BRACHE
"Mathematical discoveries, like springtime violets in the woods, have their season which no human can hasten or retard."

Janos BOLYAI
"The Analytical Engine weaves algebraic patterns, just as the Jacquard loom weaves flowers and-leaves"
Augusta Ada KING Countess of LOVELACE "An expert is someone who knows some of the worst mistakes that can be made in his subject, and how to avoid them"

Werner Karl HEISENBERG
"Analysis takes back with one hand what it gives with the other. I recoil in fear and loathing from that deplorable evil: continuous functions with no derivatives."

Charles HERMITE
"Priusquam autem ad creationem, hoc est ad finem omnis disputationis, veniamus: tentanda omnia existimo"

Johannes KEPLER

